

MEMOIRS  
OF THE  
COLLEGE OF SCIENCE

KYOTO IMPERIAL UNIVERSITY

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VOL. III, No. 10

MARCH 1919

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KYOTO



# The Spectrum Lines of Oxygen and of Nitrogen in an Intense Electric Field.

By

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(Received September 23, 1918.)

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With respect to the Stark effect in nitrogen and oxygen, only a negative result on the band spectrum of nitrogen has been obtained by Stark; and, so far as the writer is aware, no effect of an electric field on the spectrum lines of these gases has hitherto been found. In the present investigation, the experimental arrangement used for photographing the spectrum and for exciting the discharge in the vacuum tube was essentially the same as that described in the former paper relating to the Stark effect in lithium<sup>1</sup>; and a three-prism-spectrograph was used throughout the present experiment. The vacuum tube used on this occasion was the same as that stated in the previous investigation on hydrogen and helium, and the cathode was inserted in a capillary tube of hard glass. For the cathode metal, aluminium was employed in the earlier part of the experiment, but afterward tantalum was adopted as being more satisfactory, for the sputtering was far less with tantalum than with aluminium, as was noted by Dr. Takamine and Mr. Kokubu<sup>2</sup> in their investigation on the Stark effect in helium and hydrogen.

The field strength was calculated by using the data given by Stark for the Balmer lines of hydrogen in the case of the "Grobzerlegung," assuming that the proportionality between the field intensity and the amount of separation holds good even up to the strong field that was used in the present investigation.

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<sup>1</sup> Yoshida, Mem. Coll. Sci., Kyoto, 3, 161, (1918).

<sup>2</sup> Takamine and Kokubu, Proc. Tokyo Mathematico-Phys. Soc., 9, 394, (1918).

### Nitrogen.

By adopting a capillary tube of about one mm. in diameter, and increasing the field strength as much as possible, it was found that, when the gas in the tube was air, two spectrum lines at  $4110.1 \text{ \AA.U.}$  and  $4100.3 \text{ \AA.U.}$  were evidently displaced toward the violet side both in the parallel and in the perpendicular component. When the gas in the tube was replaced by oxygen or hydrogen, no trace of these two lines was detected, but when the tube was filled with nitrogen prepared chemically from sodium nitrite and ammonium sulphate, intense images of these two lines were observed on the photographic plate.

With the vacuum tube filled with nitrogen gas, Moissan and Deslandres<sup>1</sup> observed the five lines,  $4151.7$ ,  $4143.7$ ,  $4110.0$ ,  $4108.0$  and  $4100.5 \text{ \AA.U.}$ , and from their minute examination on the condition under which these lines appeared, they concluded that the lines were due to nitrogen or to an unknown gas whose chemical nature is similar to that of nitrogen.

Now in the present experiment, when the vacuum tube was filled with air or chemically prepared nitrogen, three lines at  $4151.5$ ,  $4144.0$  and  $4108.3 \text{ \AA.U.}$  besides the two lines before mentioned were detected; and the order of their intensities were nearly similar to those given by Moissan and Deslandres. Consequently it may be concluded that the two spectrum lines under consideration are the same as those observed by Moissan and Deslandres at  $4100.5$  and  $4110 \text{ \AA.U.}$

W. Hermann<sup>2</sup> and H. Wilsar<sup>3</sup> in their investigations on the Doppler effect of the canal rays of nitrogen, regarded these two spectrum lines under consideration as due to nitrogen. And there seems to be no contradiction in regarding the two spectrum lines at  $4110$  and  $4100 \text{ \AA.U.}$  as those arising from nitrogen.

Some of the photographs obtained in the present experiment are represented in Fig. 1, Fig. 2, Plate I. The maximum electric field, in Fig. 1, calculated from the separation of  $H_\gamma$  was  $7.6 \times 10^4$  volt/cm., and that in Fig. 2 calculated from the separation of  $H_\beta$  was  $16.5 \times 10^4$  volt/cm. The dispersions on the photographic plate at Balmer lines of hydrogen, and the displacements of the lines  $4100$  and  $4110 \text{ \AA.U.}$  in the field of  $16.5 \times 10^4$  volt/cm are given in the Table 1 and Table 2.

<sup>1</sup> Moissan and Deslandres, C.R., **126**, 1689, (1898).

<sup>2</sup> Hermann, Phys. Z.S., **7**, 567, (1906).

<sup>3</sup> Wilsar, Ann. d. Physik, **39**, 1251, (1912).

TABLE I.

|                               |      |      |      |
|-------------------------------|------|------|------|
| Balmer lines                  | 4861 | 4341 | 4102 |
| dispersion in Å.U.<br>per mm. | 60.0 | 39.0 | 28.0 |

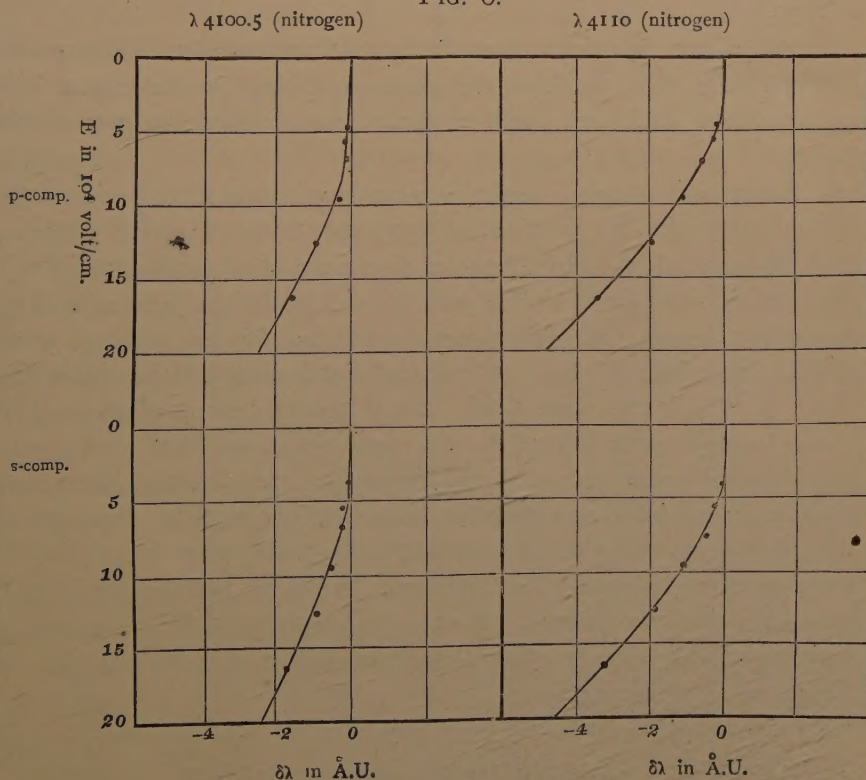
TABLE 2.

Field-strength =  $16.5 \times 10^4$  volt/cm.

|                         | 4100 Å.U. |         | 4110 Å.U. |         |
|-------------------------|-----------|---------|-----------|---------|
|                         | p-comp.   | s-comp. | p-comp.   | s-comp. |
| displacement<br>in Å.U. | -1.7      | -1.8    | -3.5      | -3.2    |

The details of the result of measurement are represented graphically in Fig. 6. Here it is worthy of note that, both in the line

FIG. 6.



4100 and in 4110 Å.U., the amount of displacement of the parallel and of the perpendicular components is equal in the limit of accuracy of measurement, the displacement of the line 4100 Å.U. being slighter than that of the other.

In a recently published paper on the Stark effect in Ca and Mg, Dr. Takamine and Mr. Kokubu<sup>1</sup> reported that the two lines at 4110.7 and 4110.3 Å.U. of unknown origin were displaced slightly toward the violet side in an intense electric field. Probably these will be the same as those described above.

Excluding the two lines above discussed, the other three observed by Moissan and Deslandres made also their appearance in the photograph as was stated before; but, so far as the present experiment is concerned, no effect of an electric field on these lines was detected.

In the region from 4000 to 5000 Å.U., many spectrum lines belonging to the negative band spectrum and the second positive band spectrum of nitrogen also appeared in the photograph, but no effect of an electric field was detected even in the strong field of  $16.5 \times 10^4$  volt/cm.

Many other spectrum lines belonging to the line spectrum of nitrogen were also visible in the photograph, but, in the region from 4000 to 5000 Å.U., no positive result was obtained on any one of them.

Lastly, on testing the region of wave-lengths longer than 5000 Å.U., it was found that the two lines at 5553 and 5688 Å.U.<sub>2</sub> were displaced evidently toward the red side, the displacement of each of these two lines being equal both in the parallel and in the perpendicular components. The amount of displacement of the former line was about 9 Å.U. in a field of  $8.6 \times 10^4$  volt/cm., and that of the latter was about 4 Å.U. in the same field. Here it must be remarked that the wave lengths given above to the two spectrum lines now under consideration were not accurate, owing to the fact that the photographic images of these two lines were diffuse and the dispersion in this region was very small with the three-prism-spectrograph used in the present experiment. And, though it is very probable that these two lines are of nitrogen at 5551.0 and 5686.3 Å.U., it is not, of course, definitely proven that these are really due to nitrogen and further study is necessary.

<sup>1</sup> Takamine and Kokubu, Mem. Coll. Sci., Kyoto, 3, 173, (1918).

### Oxygen.

In the present experiment, the spectrum lines belonging to the series spectrum and the spark spectrum or the elementary line spectrum of oxygen were examined. And some positive result was obtained on some of the series spectrum. Oxygen gas was generated by heating a mixture of potassium chlorate and manganese dioxide, then it was dried by phosphor pentoxide. For photographing the spectrum the three-prism-spectrograph described in the case of nitrogen was also used in this case for the spectrum region below  $H\beta$ . For the region above  $H\beta$ , the same spectrograph was so readjusted that it was in a state of minimum deviation at the green line of mercury. The dispersions of this readjusted spectrograph at various wave lengths are shown in the following table Table 3.

TABLE 3.

| Spectrum lines                | 4861 | 4968.5 | 5019.5 | 5329.5 | 5436 | 6157 | 6455 |
|-------------------------------|------|--------|--------|--------|------|------|------|
| dispersion in Å.U.<br>per mm. | 33   | 39     | 42     | 53     | 60   | 111  | 116  |

With this readjusted spectrograph and the former one used in the case of nitrogen, many spectrum lines belonging to the spark spectrum of oxygen were visible on the photographic plates, but no trace of displacement or separation of these lines was detected with a strong electric field of about  $10 \times 10^4$  volt/cm.

The series spectrum lines which appeared in the photograph with an exposure of about six hours, are given in Table 4. In this table the notation T.N.I. represents triplet subordinate series, and T.N.II triplet second subordinate series, and T.H. triplet principal series, and P.H. represents pair principal series.

TABLE 4.

| notation                             | T.N.II                         | T.N.I                          | T.N.II                         | T.N.I                          | T.N.II                      | T.N.I                       | P.H.       | T.H.                           |
|--------------------------------------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|-----------------------------|-----------------------------|------------|--------------------------------|
| wave length<br>measured              | 6455                           | 6157                           | 5436                           | 5329.5                         | 5019.5                      | 4968.5                      | 4368.0     | 3947.0                         |
| Runge<br>and<br>Paschen <sup>1</sup> | { 6456.287<br>54.756<br>53.900 | { 6158.415<br>56.993<br>56.198 | { 5437.041<br>35.968<br>35.371 | { 5330.835<br>29.774<br>29.162 | { 5020.31<br>19.52<br>18.96 | { 4968.94<br>68.04<br>67.58 | { 4368.466 | { 3947.759<br>47.661<br>47.480 |

<sup>1</sup> Runge and Paschen, Ann. d. Physik, 61, 641, (1897).

Among these lines, the two 4368.0 and 3947.0 Å.U. which belong to the pair principal series and the triplet principal series respectively suffered no observable influence in a field of about  $10 \times 10^4$  volt/cm. And on the line 6455 Å.U. which belongs to the second subordinate triplet series no measurable influence was detected in the field of  $6.8 \times 10^4$  volt/cm. with the present experimental condition under which the dispersion at this line was small. All the other lines which belong to the first subordinate triplet series or to the second subordinate triplet series suffered a comparatively large influence, that on the lines of the first subordinate triplet series being larger than that on the others.

The photographs reproduced in Fig. 3, Fig. 4, Fig. 5, Plate I, represent the behaviors of these lines in the electric field. The maximum electric field in the case of Fig. 3 was  $6.8 \times 10^4$  volt/cm. and that in the case of Fig. 4 was  $8.6 \times 10^4$  volt/cm. and  $8.2 \times 10^4$  volt/cm. in the case of Fig. 5. As the intensities of the series lines of oxygen were weak under the present experimental condition, an exposure of five or six hours was needed to obtain these photographs.

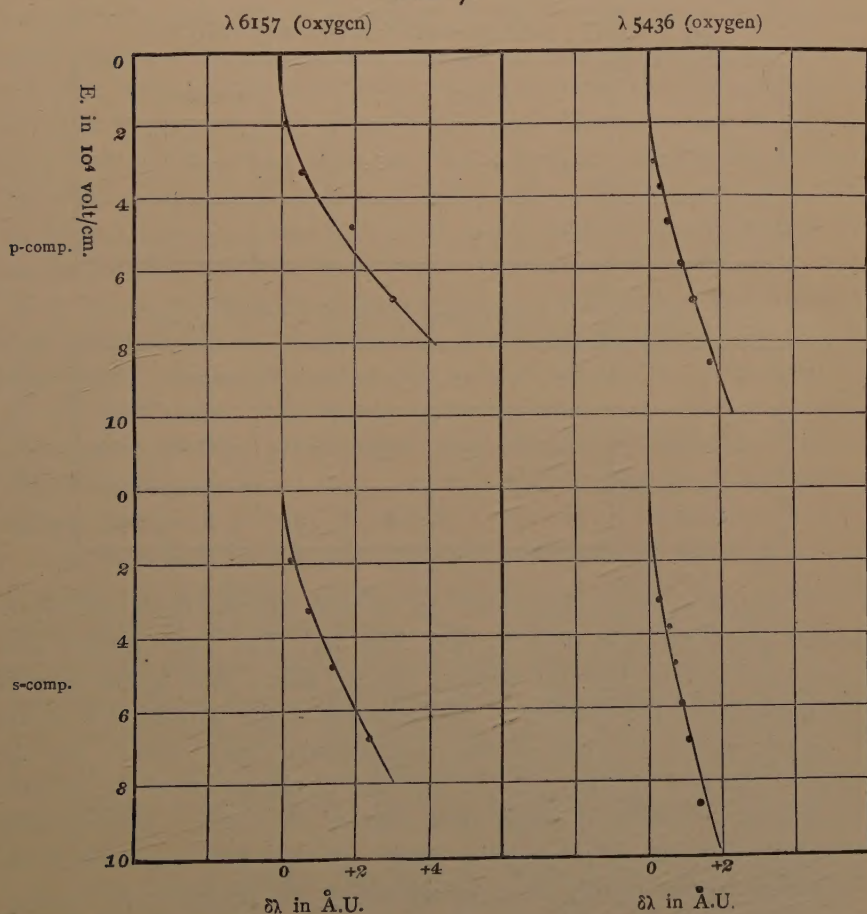
Here it must be remarked that, as the dispersion at these lines with the three-prism-spectrograph now used was small, these lines were not resolved as triplets; and therefore it was necessary in the following to regard these unresolved triplets as if they were simple spectrum lines.

*Second Subordinate Triplet Series.* Among the spectrum lines belonging to this series, the three lines denoted in Table 4 could be observed in the photograph as was shown in Fig. 3, Fig. 4, and Fig. 5, Plate I. And a positive result was obtained on the lines 5436 and 5019 Å.U. The behaviors of these lines in several electric fields are represented graphically in Fig. 7, and Fig. 9. As the field strength is increased gradually these two lines were displaced continuously toward the red side both in the parallel and perpendicular component, the amount of displacements of the parallel and of the perpendicular components being each equal in both cases. Considering the fact that the displacement of the line 5019 Å.U. is greater than that of the line 5436 Å.U., and that no effect was observed on the line 6455 Å.U., there seems to be no contradiction to conclude that the displacement of a line in this series is larger as the term number of the series increases. All these behaviors of the spectrum lines belonging to the second subordinate triplet series of oxygen are quite

similar to those of the second subordinate series spectrum of helium and parhelium<sup>1</sup>.

*First Subordinate Triplet Series.* Among the spectrum lines of this series, the three lines 6157, 5329.5 and 4968.5 Å.U. could be observed on the photographic plate, and a positive result was obtained on all these lines, as was seen in Fig. 3 and Fig. 5, Plate I. The effect of an electric field on the spectrum line 6157 Å.U. was the displacement toward the red side, and the result of the measurement is represented graphically in Fig. 7. Here the displacement of the parallel component is somewhat larger than that of the perpendicular

FIG 7.



<sup>1</sup> Takamine and Yoshida, Mem. Coll. Sci., Kyoto, **2**, 325, (1917); and Takamine and Kokubu, Mem. Coll. Sci., Kyoto, **3**, 81, (1918).

one, but this may perhaps be due to an experimental error. For the dispersion at this line was very small with the present experimental arrangement, and the photographic image of this line was more intense and consequently appeared more broadened in the perpendicular component than in the parallel one. And therefore the displacements of the parallel and the perpendicular components may be regarded as of nearly equal amount.

Next the electric effect on the lines 4968.5 and 5329.5 Å.U. is shown in Fig. 5, Plate I, and the result of the measurement at the maximum electric field applied are tabulated in the following table Table 5.

TABLE 5.

4968.5 Å.U., field-strength =  $8.16 \times 10^4$  volt/cm.

|                         | p-comp. |      |       | s-comp. |      |       |
|-------------------------|---------|------|-------|---------|------|-------|
| $\delta\lambda$ in Å.U. | -15.6   | +2.3 | +15.9 | +15.9   | +2.8 | +16.9 |
| Intensity               | 1       | 1    | 1     | 2       | 2    | 2     |

5329.5 Å.U., field-strength =  $8.16 \times 10^4$  volt/cm.

|                         | p-comp. |       |       | s-comp. |       |      |
|-------------------------|---------|-------|-------|---------|-------|------|
| $\delta\lambda$ in Å.U. | -30.0   | -10.0 | +10.8 | -30.1   | -11.6 | +7.6 |
| Intensity               | 1       | 4     | 8     | 2       | 5     | 10   |

The behaviors of these lines in several electric fields are represented graphically in Fig. 8, and Fig. 10. The lines 4968.5 and 5329.5 Å.U. were displaced toward the red side. And in the case of the former line, the amount of displacement of the parallel component is nearly equal to that of the perpendicular one. This is not the case on the line at 5329.5 Å.U., and the displacement of the parallel component is somewhat larger than that of the other. Judging from the fact that the photographic image of this line appeared more intense and consequently more broadened in the perpendicular component than in the parallel one, the inequality of the displacements of the two components may be ascribed to an experimental error. And

consequently we may conclude that the displacements of the parallel and of the perpendicular components are nearly equal respectively in the case of the three spectrum lines before mentioned.

Next, in Fig. 5, Plate I, two spectrum lines will be seen, so to speak quite isolated, in the immediate violet side of the line 5329.5 Å.U.; also in the immediate violet side of 4968.5 Å.U., two very weak isolated lines are visible exactly in the corresponding position as before. The behavior of these lines in several electric fields are represented graphically in Fig. 8, and Fig. 10, in conjunction with those of 4968.5 and 5329.5 Å.U. In order to determine whether these isolated spectrum lines were due to oxygen or some other impurities, the writer examined it more closely. On measuring the wave lengths of all the spectrum lines which were seen on the photograph, it was found that, excepting the lines which are due to oxygen, most of the other lines are of hydrogen and nitrogen, and a few are of mercury and sodium. Of these four elements, the latter two have no spectrum lines in the neighbourhood of the isolated spectrum lines before mentioned. The other two elements, especially hydrogen, have some spectrum lines which might cause the appearance of the isolated spectrum lines before mentioned. As these weak lines of hydrogen or nitrogen, which might be considered as the continuation of the isolated spectrum lines in the region of no electric effect, was visible on the photograph, the writer experimented with hydrogen or nitrogen alone. And it was confirmed that none of the questionable spectrum lines of hydrogen and nitrogen showed any electrical effect. Consequently we may consider that the isolated spectrum lines before mentioned are due to oxygen.

With regard to the four isolated spectrum lines above discussed, it seems, to the writer, to be convenient to consider them as isolated components of the lines 5329.5 and 4968.5 Å.U., as in the case of helium<sup>1</sup> and lithium<sup>2</sup>. All these isolated components appeared on the immediate violet sides of the lines 4968.5 and 5329.5 Å.U., as in the case of other elements. Moreover the isolation of the isolated components of 4968.5 Å.U. is of smaller amount than that of the isolated components of 5329.5 Å.U. This is similar to the behavior of the

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<sup>1</sup> Takamine and Yoshida, *Mem. Coll. Sci., Kyoto*, **2**, 325, (1917); Takamine and Kokubu, *Ibid*, **3**, 81, (1918); Takamine and Kokubu, *Proc. Tokyo Math. Phys. Soc.*, **9**, 394, (1918).

<sup>2</sup> Yoshida, *Mem. Coll. Sci., Kyoto* **3**, 161, (1918).

isolated components of the spectrum lines of helium and parhelium<sup>1</sup>, of which the isolation of an isolated component of a spectrum line of the first subordinate series become smaller as the term number of the series increases.

In the previous paper dealing with the Stark effect in lithium<sup>2</sup>, it was stated that the lithium line  $4602 \text{ \AA.U.}$  had a so-called isolated component on its immediate violet side, and the fact observed by Saunders<sup>3</sup> that this line became double when broadened was explained by the appearance of an isolated component of this spectrum line in an electric field. Recently Merton<sup>4</sup> investigated the broadening of helium lines by condensed spark discharges, and found a close agreement between the broadening and the electrical resolution of helium lines. Especially interesting is Merton's result, that the isolated components in the electrical resolution were observed in the broadened lines. In the present case, some isolated components of oxygen lines were found in an electric field as stated before; and an investigation on the broadening of oxygen lines will be of much interest.

*Band Spectrum Lines of Oxygen.* In the present experiment, only two negative band spectra at  $563\text{--}555 \mu\mu$  and  $529\text{--}520 \mu\mu$  of oxygen were visible on the photographic plate. The intensity of these band spectra was very much reduced in the Crookes' dark space where the electric field was much stronger than in the other portion of the discharge tube. Owing to their weak intensities the writer could not ascertain whether the wave lengths of the band spectrum lines of oxygen were affected by an electric field or not.

The intensities of the spectrum lines of the triplet series of oxygen increase, contrary to the case of band spectra, in the Crookes' dark space, and they become stronger continuously as we pass from the region of weaker electric field to the stronger one. Next with regard to the spark spectrum lines of oxygen, they behave similarly as the band spectrum of oxygen. But the diminution of the intensities of the spark lines in the Crookes dark space is slighter than in the other case.

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<sup>1</sup> Takamine and Kokubu, Mem. Coll. Sci., Kyoto, **3**, 81, (1918).

<sup>2</sup> Yoshida, loc. cit.

<sup>3</sup> Saunders, Astrophys. J., **20**, 188, (1904).

<sup>4</sup> Merton, Nature, **101**, 279, (1918).

### Summary.

1. The Stark effect on the spectrum lines of nitrogen and oxygen was investigated, and a positive result was obtained on the lines at 4100 and 4110 Å.U. of nitrogen, and on some of the lines belonging to the first and second subordinate triplet series of oxygen.

2. Excepting a few ambiguous cases, the amount of the displacement or of the separation of each of all these lines is nearly equal respectively in both the parallel and the perpendicular components.

3. The behavior of the lines of the second subordinate triplet series of oxygen in the electric field is similar to that of the lines belonging to the second subordinate series of helium and parhelium.

4. The effect on the spectrum lines of the first subordinate triplet series of oxygen is larger than that on the second subordinate triplet series.

5. Each of the lines 5329.5 and 4968.5 Å.U. of the first subordinate triplet series have two isolated components respectively in its immediate violet side. The isolation of the said components become smaller as the term number of the series increases. This is exactly similar to that of the isolated components of helium and parhelium

In conclusion the writer's sincere thanks are due to Prof. T. Mizuno for his interest in the research.

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FIG. 8.  
 $\lambda$  4968.5 (oxygen).

FIG. 9.  
 $\lambda$  5019 (oxygen).

FIG 10.

$\lambda$  5330 (oxygen).

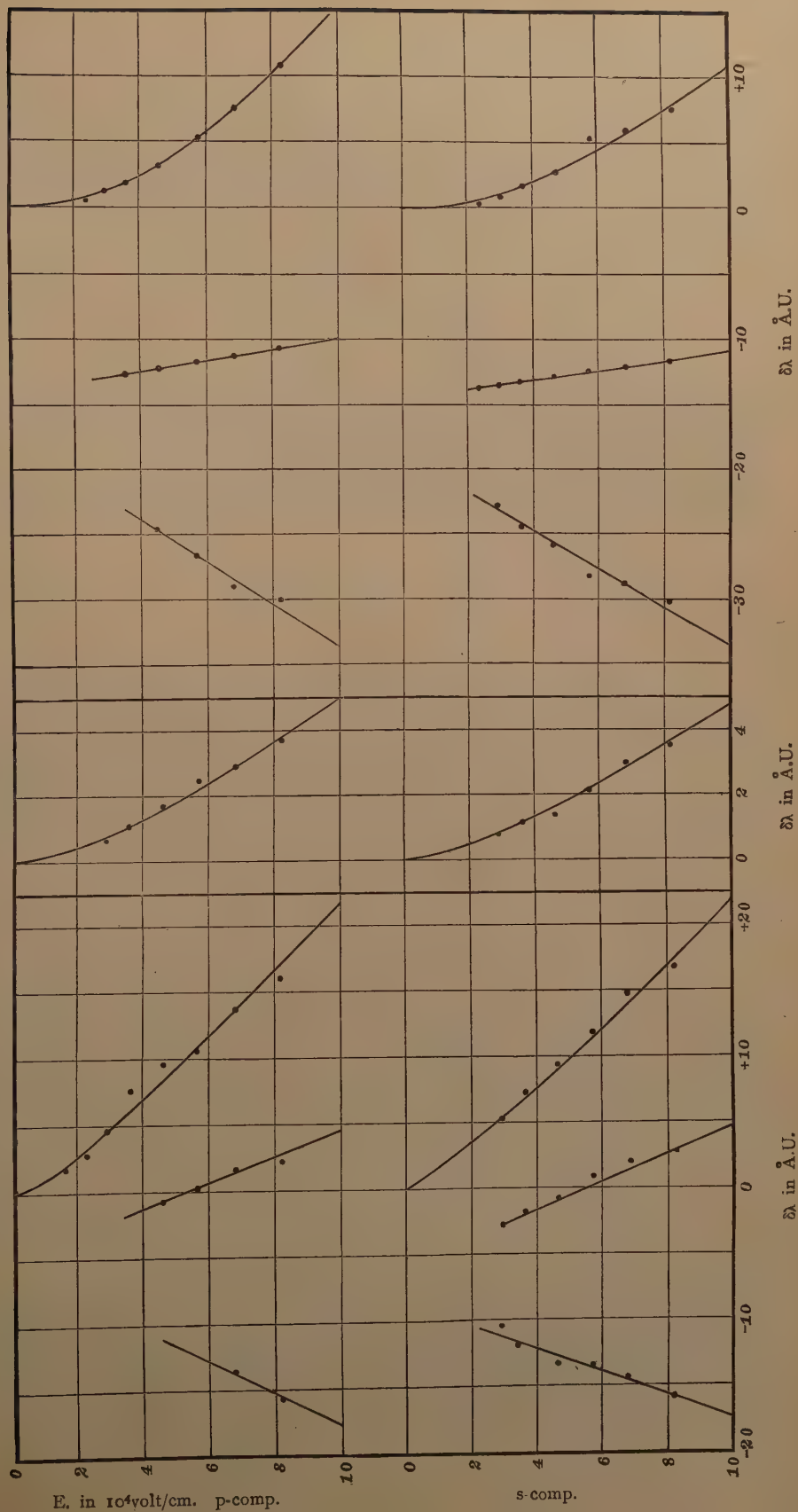
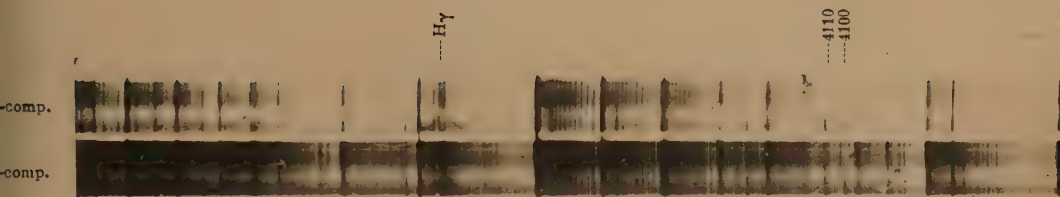


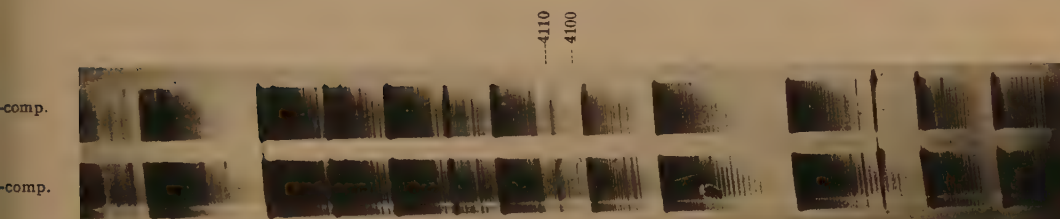


Fig. 1. Nitrogen



Magnification 7.  $E_{\max.} = 7.6 \times 10^4 \text{ volt/cm.}$

Fig. 2. Nitrogen



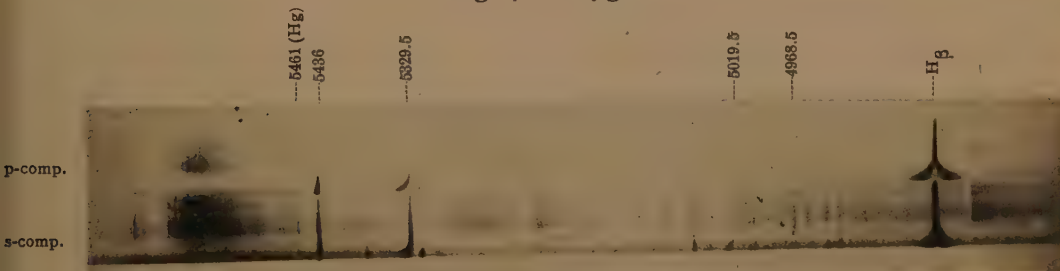
Magnification 6.7.  $E_{\max.} = 16.5 \times 10^4 \text{ volt/cm.}$

Fig. 3. Oxygen



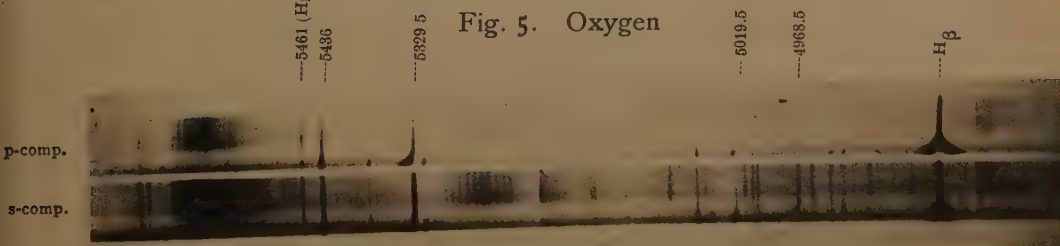
Magnification 6.1.  $E_{\max.} = 6.8 \times 10^4 \text{ volt/cm.}$

Fig. 4. Oxygen



Magnification 6.4.  $E_{\max.} = 8.6 \times 10^4 \text{ volt/cm.}$

Fig. 5. Oxygen



Magnification 6.4.  $E_{\max.} = 8.2 \times 10^4 \text{ volt/cm.}$



## On Congruences. IV.

By

Masazo Sono.

(Received Nov. 25, 1918.)

The present paper is intended, in the main, to discuss the ideals of a proper ring which contains at least two elements, distinct from 0, whose product is equal to 0, under the assumption that corresponding to any ideal, distinct from the 0-ideal, the ring possesses a chief-composition-series containing the ideal for the last term. In § 2 we show that in such a ring the maximal ideals are finite in number and that  $\mathfrak{P}^e = \mathfrak{P}^{e+1}$  for each maximal ideal  $\mathfrak{P}$ , and for a certain exponent  $e$ , in § 3 we treat the converse, in § 5 we consider the resolution of ideals in such a ring, in § 6 we add a word about the product of the ideals of a quotient ring, and in §§ 7, 8 we give examples of resolvability. The result obtained in § 3 requires the correction of the theorems of Congr. II, § 28 and Congr. III, § 8: so that it is given in § 4.

For the sake of brevity the previous papers<sup>1</sup> on congruences are here denoted by Congr., Congr. II and Congr. III respectively.

### A Proper Ring which contains at least Two Elements whose Product is Equal to 0.

§ 1. Let  $\mathfrak{R}$  be a proper ring; suppose that corresponding to an ideal  $\mathfrak{A}$  of  $\mathfrak{R}$ , there exists a chief-composition-series of  $\mathfrak{R}$  which contains  $\mathfrak{A}$  for the last term. Then, as shown in §§ 4, 5 of Congr. III, the maximal ideals containing  $\mathfrak{A}$  are finite in number. If  $\mathfrak{A}$  is contained in a single maximal ideal, say  $\mathfrak{P}$ ,  $\mathfrak{A}$  contains a power of  $\mathfrak{P}$ . If, on the contrary,  $\mathfrak{A}$  is contained in just  $\nu$  maximal ideals,  $\mathfrak{A}$  is resolvable into  $\nu$  factors prime to one another, each of which contains

<sup>1</sup> On Congruences, these Memoirs, 2, 203 (1917);  
On Congruences, II, these Memoirs, 3, 113 (1918);  
On Congruences, III, these Memoirs, 3, 189 (1918).

a power of one of the maximal ideals; and this can be done in one way only.

But in the present paper our discussion is limited to the ideals of such a proper ring as, corresponding to any given ideal distinct from the  $\mathfrak{o}$ -ideal, possesses a chief-composition-series containing the ideal for the last term: so that the result above described is always applicable.

§ 2. We now suppose that a proper ring  $\Re$  contains at least two elements, distinct from  $\mathfrak{o}$ , whose product is equal to  $\mathfrak{o}$ .

Let  $A, B$  be such elements, i.e.

$$AB = \mathfrak{o},$$

while both are distinct from  $\mathfrak{o}$ . Then the principal idea  $(A)$  is distinct from  $\Re$ . For, otherwise, an element  $R$  could be so chosen that  $AR = 1$ ; and hence we should have  $B = B \cdot AR = \mathfrak{o}$ , contrary to our assumption. Similarly the ideal  $(B)$  is also distinct from  $\Re$ . Therefore, as stated in § 1,  $(A)$  and  $(B)$  either contain powers of maximal ideals or are resolvable into factors which respectively contain powers of distinct maximal ideals. So that, while  $AB = \mathfrak{o}$ , the product  $(AB)$  either contains a power of a maximal ideal, say  $\mathfrak{P}^e$ , or may be expressed in the form:

$$(AB) = \mathfrak{L}_1 \mathfrak{L}_2 \dots \mathfrak{L}_v,$$

where  $\mathfrak{L}_1, \mathfrak{L}_2, \dots, \mathfrak{L}_v$  are ideals respectively containing powers of distinct maximal ideals  $\mathfrak{P}_1, \mathfrak{P}_2, \dots, \mathfrak{P}_v$ .

In the former case, the power  $\mathfrak{P}^e$  must be the  $\mathfrak{o}$ -ideal; hence  $\Re$  has no other maximal ideals distinct from  $\mathfrak{P}$  [Congr. II, § 4]. It will be seen in §§ 7, 8 that this case is existent.

In the latter, no powers of the  $\mathfrak{P}$ 's are the  $\mathfrak{o}$ -ideal; because otherwise  $\Re$  might have a single maximal ideal only. Therefore all of the following:  $\mathfrak{L}_1, \mathfrak{L}_2, \dots, \mathfrak{L}_v$  are distinct from the  $\mathfrak{o}$ -ideal. If  $\mathfrak{P}$  were another maximal ideal distinct from  $\mathfrak{P}_1, \mathfrak{P}_2, \dots, \mathfrak{P}_v$ , then  $\mathfrak{P}$  would be prime to all of  $\mathfrak{L}_1, \mathfrak{L}_2, \dots, \mathfrak{L}_v$ , and consequently to their product  $\mathfrak{L}_1 \mathfrak{L}_2 \dots \mathfrak{L}_v$  [Congr. II, § 4], while  $\mathfrak{L}_1 \mathfrak{L}_2 \dots \mathfrak{L}_v = \mathfrak{o}$ . Therefore there are no other maximal ideals distinct from  $\mathfrak{P}_1, \mathfrak{P}_2, \dots, \mathfrak{P}_v$ .

And moreover we have

$$\begin{aligned} \mathfrak{P}_1 \mathfrak{L}_1 &= (\mathfrak{P}_1 \mathfrak{L}_1, \mathfrak{L}_1 \mathfrak{L}_2 \dots \mathfrak{L}_v) \\ &= \mathfrak{L}_1 (\mathfrak{P}_1, \mathfrak{L}_2 \dots \mathfrak{L}_v) \\ &= \mathfrak{L}_1 \Re = \mathfrak{L}_1. \end{aligned}$$

But  $\mathfrak{L}_1$  contains a power of  $\mathfrak{P}_1$ , say  $\mathfrak{P}_1^{e_1}$ : so that  $\mathfrak{P}_1$  contains  $\mathfrak{L}_1$  [Congr. II, § 4]. Hence  $\mathfrak{P}_1^{e_1}$  contains  $\mathfrak{L}_1 \mathfrak{P}_1^{e_1-1}$ , which is equal to  $\mathfrak{L}_1$ . Therefore  $\mathfrak{L}_1 = \mathfrak{P}_1^{e_1}$ , and hence

$$\mathfrak{P}_1^{e_1} = \mathfrak{P}_1^{e_1+1}.$$

Similarly we have, for a certain exponent  $e_i$ ,

$$\mathfrak{P}_i^{e_i} = \mathfrak{P}_i^{e_i+1} \quad (i=2, 3, \dots \nu).$$

and

$$\mathfrak{P}_1^{e_1} \mathfrak{P}_2^{e_2} \dots \mathfrak{P}_\nu^{e_\nu} = 0.$$

It has already been shown by the example in Congr. II, § 9 that this case is also existent. And we have the

**THEOREM:** *If a proper ring  $\mathfrak{R}$  contains at least two elements, distinct from 0, whose product is equal to 0, then (i) the maximal ideals of  $\mathfrak{R}$  are finite in number, (ii) if the number is one, a certain power of the maximal ideal is the 0-ideal, and (iii) if it is more than one,*

$$\mathfrak{P}^e = \mathfrak{P}^{e+1} \neq 0,$$

*for each maximal ideal  $\mathfrak{P}$  and for a certain exponent  $e$ .*

N. B. Herein it is assumed that, corresponding to a given ideal, distinct from the 0-ideal, there exists a chief-composition-series which contains the ideal for the last term.

§ 3. To treat the converse, assume that a proper ring  $\mathfrak{R}$  contains a maximal ideal  $\mathfrak{P}$  for which  $\mathfrak{P}^e = \mathfrak{P}^{e+1}$  for a certain exponent  $e$ .

If  $\mathfrak{P}^e = 0$ , evidently  $\mathfrak{R}$  contains two elements, distinct from 0, whose product is equal to 0; and moreover  $\mathfrak{P}$  is the only maximal ideal of  $\mathfrak{R}$  [Congr. II, § 4].

Next, to consider the case in which  $\mathfrak{P}^e \neq 0$ , take an element  $A$ , distinct from 0, of  $\mathfrak{P}^e$ . Then the principal ideal  $(A)$  either contains a power of  $\mathfrak{P}$  or is resolvable into factors prime to each other:

$$(A) = \mathfrak{A}\mathfrak{M},$$

where  $\mathfrak{A}$  is an ideal containing a power of  $\mathfrak{P}$ .

In the former case, evidently  $(A)$  must contain  $\mathfrak{P}^e$ , and consequently coincides with  $\mathfrak{P}^e$ : so that

$$A\mathfrak{P} = \mathfrak{P}^{e+1} = \mathfrak{P}^e = (A).$$

Hence we have

$$AP = A,$$

$P$  being an element of  $\mathfrak{P}$ , or

$$A(P-1)=0,$$

while  $P-1$  is distinct from 0; because  $P$  belongs to  $\mathfrak{P}$ , and consequently may not be equal to 1.

In the latter,  $\mathfrak{A}$  must contain  $\mathfrak{P}^e$ , and hence  $\mathfrak{A}\mathfrak{M}$  contains  $\mathfrak{P}^e\mathfrak{M}$ . But  $\mathfrak{P}^e\mathfrak{M}$  contains  $(A)$ ; because, since  $(\mathfrak{P}^e, \mathfrak{M})=\mathfrak{R}$ , the product  $\mathfrak{P}^e\mathfrak{M}$  is the cross-cut of  $\mathfrak{P}^e$  and  $\mathfrak{M}$ , both of which contain  $A$  [Congr. II, § 5]. Therefore  $(A)$  and  $\mathfrak{P}^e\mathfrak{M}$  are identical: so that

$$A\mathfrak{P}=\mathfrak{P}^{e+1}\mathfrak{M}=\mathfrak{P}^e\mathfrak{M}=(A),$$

whence, as above, we have

$$A(P-1)=0,$$

where  $P$  is an element of  $\mathfrak{P}$ .

Therefore  $\mathfrak{R}$  contains at least two elements, distinct from 0, whose product is equal to 0. And we have the

**THEOREM:** *If there exists in a proper ring  $\mathfrak{R}$  a maximal ideal  $\mathfrak{P}$ , for which  $\mathfrak{P}^e=\mathfrak{P}^{e+1}$  for a certain exponent  $e$ ,  $\mathfrak{R}$  contains at least two elements, distinct from 0, whose product is equal to 0.*

If, therefore, in a proper ring  $\mathfrak{R}$  the product of two elements is distinct from 0, unless at least one of the factors is equal to 0, we have

$$\mathfrak{P}^e \neq \mathfrak{P}^{e+1}$$

for any maximal ideal  $\mathfrak{P}$  and for any exponent  $e$ . Hence the theorem of Congr. III, § 8 and that of Congr. II, § 28 must be corrected: the former will be given as the theorem and the latter as the corollary in the next article.

### Resolvability of Ideals.

§ 4. Let  $\mathfrak{R}$  be a proper ring subject to the following conditions:

(1) The product of two elements of  $\mathfrak{R}$  is distinct from 0, unless at least one of the factors is equal to 0;

(2) Corresponding to any given ideal  $\mathfrak{A}$ , distinct from the 0-ideal, of  $\mathfrak{R}$ , there exists a chief-composition-series of  $\mathfrak{R}$  which contains  $\mathfrak{A}$  for the last term.

Then we have the

**THEOREM:** *In order that every composite ideal of  $\mathfrak{R}$  may be uniquely resolvable into prime factors, it is necessary and sufficient that*

for every maximal ideal  $\mathfrak{P}$  of  $\mathfrak{R}$  the following condition should hold: an ideal of  $\mathfrak{R}$  which contains  $\mathfrak{P}^2$  is either  $\mathfrak{P}$  or  $\mathfrak{P}^2$ .

The proof here is repeated for the sake of completeness, slight as the correction is.

To prove the necessity of the condition we assume that every composite ideal is uniquely resolvable into prime factors; let  $\mathfrak{P}$  be a maximal ideal of  $\mathfrak{R}$ . Then, by condition (1) for the ring  $\mathfrak{R}$ ,  $\mathfrak{P}^2$  is distinct from the 0-ideal, and hence, by condition (2), there is a chief-composition-series containing  $\mathfrak{P}^2$  for the last term. And moreover  $\mathfrak{P}^2$  is distinct from  $\mathfrak{P}$ , as has been already proved. Therefore the chief-composition-series consists either of three terms or of more than three terms. Namely, it is either

$$\mathfrak{R}, \mathfrak{P}, \mathfrak{P}^2,$$

or

$$\mathfrak{R}, \mathfrak{P}, \mathfrak{A}_1, \mathfrak{A}_2, \dots, \mathfrak{A}_n, \mathfrak{P}^2 \quad (n \geq 1).$$

If the latter case happened, the  $\mathfrak{A}$ 's would be all prime, and hence at least one of the ideals containing  $\mathfrak{P}^3$  would be resolvable into prime factors in more than one way, as seen from the results obtained in Congr. II, §§ 11-15. Therefore an ideal containing  $\mathfrak{P}^2$  must be either  $\mathfrak{P}$  or  $\mathfrak{P}^2$  [Congr. II, § 4].

Next, to prove the sufficiency of the condition we assume that it is fulfilled. Then the maximal ideals are all prime, but the others all composite [Congr. III, § 7]; and also an ideal is resolvable into the product of the maximal ideals [Congr. III, § 6]. Therefore a composite ideal  $\mathfrak{A}$  may be reduced to the form

$$\mathfrak{P}_1^{e_1} \mathfrak{P}_2^{e_2} \dots \mathfrak{P}_\nu^{e_\nu},$$

where  $\mathfrak{P}_1, \mathfrak{P}_2, \dots, \mathfrak{P}_\nu$  are distinct prime ideals.

It is clear that a prime ideal dividing  $\mathfrak{A}$  must be one of the  $\mathfrak{P}$ 's, and also that if two resolutions are possible the same prime factors must occur in both: so that the only admissible supposition is

$$\mathfrak{P}_1^{e_1} \mathfrak{P}_2^{e_2} \dots \mathfrak{P}_\nu^{e_\nu} = \mathfrak{P}_1^{e'_1} \mathfrak{P}_2^{e'_2} \dots \mathfrak{P}_\nu^{e'_\nu},$$

whence we have

$$\mathfrak{P}_i^{e_i} = \mathfrak{P}_i^{e'_i} \quad (i=1, 2, \dots, \nu) \quad [\text{Congr. III, § 5}];$$

and hence

$$e_i = e'_i \quad (i=1, 2, \dots, \nu);$$

because otherwise  $\mathfrak{P}_i^{e_i}$  and  $\mathfrak{P}_i^{e'_i}$  would be distinct [§ 3]. So that the two resolutions are identical.

**Cor.** Let  $\mathfrak{R}$  be a proper ring such that the product of two elements, distinct from 0, is not equal to 0, and moreover every ideal distinct from the 0-ideal is of finite norm. Then in order that every composite ideal of  $\mathfrak{R}$  may be uniquely resolvable into prime factors, it is necessary and sufficient that for every maximal ideal  $\mathfrak{P}$  the equation

$$n(\mathfrak{P}^2) = [n(\mathfrak{P})]^2$$

should hold.

This follows from the above theorem and the result obtained in Congr. II, § 8.

§ 5. The resolvability of an ideal of a proper ring which contains at least two elements, distinct from 0, whose product is equal to 0. If  $\mathfrak{R}$  is such a ring, the maximal ideals are finite in number; and if  $\mathfrak{P}$  is one of them, for a certain exponent  $e$ , the power  $\mathfrak{P}^e$  coincides either with the 0-ideal or with  $\mathfrak{P}^{e+1}$ , while  $\mathfrak{P}^e$  is not the 0-ideal.

(1) First taking the former case, we treat it under the assumption that  $e > 2$ ; because, if  $e = 2$ , the ideals of  $\mathfrak{R}$  are all prime.

(i) If a chief-composition-series with the last term  $\mathfrak{P}^2$  consists of three terms:

$$\mathfrak{R}, \mathfrak{P}, \mathfrak{P}^2,$$

we have

$$\mathfrak{P}^n = (\pi)^n, \mathfrak{P}^{n+1},$$

and consequently

$$\mathfrak{P} = (\pi), \mathfrak{P}^n,$$

where  $\pi$  is an element of  $\mathfrak{P}$ , which does not belong to  $\mathfrak{P}^2$  [Congr. II, § 9]. But, in the present case,  $\mathfrak{P}^e = 0$ ; therefore  $\mathfrak{P} = (\pi)$ , and  $\pi^e = 0$ . And moreover the ideals of  $\mathfrak{R}$  are given by the following  $e-1$ :

$$(\pi), (\pi)^2, \dots (\pi)^{e-1};$$

because all the ideals must be contained in  $\mathfrak{P}$ .

(ii) If, on the contrary, the series consists of more than three terms, taking an element  $P$  of  $\mathfrak{P}$  which does not belong to  $\mathfrak{P}^2$  we have

$$((P), \mathfrak{P}^2) \mathfrak{P}^{e-2} = P \mathfrak{P}^{e-2}.$$

But, when  $\mathfrak{P}^e = 0$ , an ideal which is not contained in  $\mathfrak{P}^2$  is prime; because, when  $\mathfrak{P}^e = 0$ , the ideals of  $\mathfrak{R}$  are contained in  $\mathfrak{P}$ , and hence the product of the ideals must be contained in  $\mathfrak{P}^2$ . Therefore, both  $((P), \mathfrak{P}^2)$  and  $(P)$  are prime; if these two are distinct, the resolution

of  $P\mathfrak{P}^{e-2}$  is not unique. If, on the contrary,  $((P), \mathfrak{P}^2) = (P)$ , the elements of  $\mathfrak{P}^2$  are contained in  $(P)$ , and consequently may be expressed in the form  $PR$ . In this,  $R$  must belong to  $\mathfrak{P}$ ; because, otherwise, the ideal  $(PR)$  would coincide with  $(P)$ , contrary to our assumption that  $P$  does not belong to  $\mathfrak{P}^2$ . If, conversely,  $R$  belongs to  $\mathfrak{P}$ , the product  $PR$  is an element of  $\mathfrak{P}^2$ . Therefore, if  $((P), \mathfrak{P}^2) = (P)$ , we have

$$\mathfrak{P}^2 = P\mathfrak{P},$$

which shows that the resolution of  $\mathfrak{P}^2$  is not unique.

(2) Next, to consider the case in which the number of the maximal ideals is more than one, let  $\mathfrak{P}$  be one of them. Then, as stated above,  $\mathfrak{P}^e = \mathfrak{P}^{e+1}$  for a certain exponent  $e$ . If  $e = 1$ , an ideal containing a power of  $\mathfrak{P}$  is  $\mathfrak{P}$  itself; therefore we need only consider the case in which  $e > 1$ .

(i) If there are no ideals, distinct from  $\mathfrak{P}$  and  $\mathfrak{P}^2$ , which contain  $\mathfrak{P}^2$ , an ideal containing a power of  $\mathfrak{P}$  is a power of  $\mathfrak{P}$  [Congr. II, § 10], and hence is divisible by no other prime ideals distinct from  $\mathfrak{P}$ .

(ii) In the opposite case, taking an element  $P$  of  $\mathfrak{P}$  which does not belong to  $\mathfrak{P}^2$  we have

$$((P), \mathfrak{P}^2) \mathfrak{P}^{e-1} = \mathfrak{P}^e,$$

showing that  $\mathfrak{P}^e$  is divisible by the prime ideal  $((P), \mathfrak{P}^2)$  distinct from  $\mathfrak{P}$ .

§ 6. Before giving examples which show the existence of the cases discussed in the last article, we add a word about the product of the ideals of a quotient ring.

Let  $\mathfrak{M}$  be an ideal of a ring  $\mathfrak{R}$ , and  $\mathfrak{A}, \mathfrak{B}$  other ideals of  $\mathfrak{R}$  which contain  $\mathfrak{M}$ . Then the quotients  $\mathfrak{A}/\mathfrak{M}$  and  $\mathfrak{B}/\mathfrak{M}$  are ideals of the quotient ring  $\mathfrak{R}/\mathfrak{M}$  [cf. Congr. § 9], and the elements of their product are given by the form

$$\sum \alpha\beta \pmod{\mathfrak{M}},$$

where  $\alpha, \beta$  are elements of  $\mathfrak{A}/\mathfrak{M}$  and  $\mathfrak{B}/\mathfrak{M}$  respectively. If  $\mathfrak{C}$  denotes the ideal of  $\mathfrak{R}$ , whose quotient as regards  $\mathfrak{M}$  is the product  $\frac{\mathfrak{A}}{\mathfrak{M}} \cdot \frac{\mathfrak{B}}{\mathfrak{M}}$ , the elements of  $\mathfrak{C}$  are given by the form

$$\mathfrak{M} + \sum \alpha\beta,$$

where  $M$  is an element of  $\mathfrak{M}$  [cf. Congr. § 9]; hence  $\mathfrak{C}$  is contained in the ideal  $(\mathfrak{M}, \mathfrak{AB})$ . But the elements of  $(\mathfrak{M}, \mathfrak{AB})$  may be expressed in the form

$$M + \Sigma AB,$$

where  $A, B, M$  are elements of  $\mathfrak{A}, \mathfrak{B}, \mathfrak{M}$  respectively; and

$$\Sigma AB \equiv \Sigma a\beta \pmod{\mathfrak{M}}.$$

Therefore  $(\mathfrak{M}, \mathfrak{AB})$  is contained in  $\mathfrak{C}$ : so that  $(\mathfrak{M}, \mathfrak{AB})$  and  $\mathfrak{C}$  are identical. Hence we have the

**THEOREM:** *Let  $\mathfrak{M}$  be an ideal of a ring  $\mathfrak{R}$ , and  $\mathfrak{A}, \mathfrak{B}$  other ideals of  $\mathfrak{R}$  which contain  $\mathfrak{M}$ . Then*

$$\frac{\mathfrak{A}}{\mathfrak{M}} \cdot \frac{\mathfrak{B}}{\mathfrak{M}} = \frac{(\mathfrak{AB}, \mathfrak{M})}{\mathfrak{M}}.$$

If  $\mathfrak{M}$  contains a power of a maximal ideal, say  $\mathfrak{P}^e$  ( $e > 1$ ), we have

$$\left(\frac{\mathfrak{P}}{\mathfrak{M}}\right)^e = \frac{(\mathfrak{P}^e, \mathfrak{M})}{\mathfrak{M}} = \frac{\mathfrak{M}}{\mathfrak{M}},$$

while  $\mathfrak{M}/\mathfrak{M}$  is the o-ideal of  $\mathfrak{R}/\mathfrak{M}$ .

Next, suppose that

$$\mathfrak{M} = \mathfrak{M}_1 \mathfrak{M}_2 \dots \mathfrak{M}_v,$$

where  $\mathfrak{M}_1, \mathfrak{M}_2, \dots, \mathfrak{M}_v$  are ideals respectively containing powers of distinct maximal ideals  $\mathfrak{P}_1, \mathfrak{P}_2, \dots, \mathfrak{P}_v$ .

Then

$$\left(\frac{\mathfrak{P}_i}{\mathfrak{M}}\right)^n = \frac{(\mathfrak{P}_i^n, \mathfrak{M})}{\mathfrak{M}} = \frac{(\mathfrak{P}_i^n, \mathfrak{M}_i)}{\mathfrak{M}},$$

since

$$(\mathfrak{P}_i^n, \mathfrak{M}) = (\mathfrak{P}_i^n, \mathfrak{M}_1 \mathfrak{M}_2 \dots \mathfrak{M}_v) = (\mathfrak{P}_i^n, \mathfrak{M}_i) \quad [\text{Congr. III, § 6}].$$

If, therefore,  $\mathfrak{P}_i^{e_i}$  is the lowest power of  $\mathfrak{P}_i$  which is contained in  $\mathfrak{M}_i$ , we have

$$\left(\frac{\mathfrak{P}_i}{\mathfrak{M}}\right)^{e_i+n} = \frac{\mathfrak{M}_i}{\mathfrak{M}} \quad n \geq 0; \quad i = 1, 2, \dots, v,$$

while

$$\left(\frac{\mathfrak{P}_i}{\mathfrak{M}}\right)^{e_i-1} \neq \frac{\mathfrak{M}_i}{\mathfrak{M}}.$$

§ 7. Now let  $\mathfrak{R}$  be a set of all possible polynomials of  $x, y$ , whose coefficients are numbers of a field  $\mathcal{Q}$ . Then evidently  $\mathfrak{R}$  is a

proper ring, and the ideal<sup>1</sup>  $(x, y)$  derived from  $(x)$  and  $(y)$  is maximal.

Putting  $\mathfrak{P} = (x, y)$  we have

$$\mathfrak{P}^2 = (x^2, xy, y^2),$$

$$((x), \mathfrak{P}^2) = (x, y^2),$$

$$((x), \mathfrak{P}^e) = (x, y^e) \quad (e > 2),$$

the last two of which are distinct. Consequently quotients  $((x), \mathfrak{P}^2)/\mathfrak{P}^e$  and  $((x), \mathfrak{P}^e)/\mathfrak{P}^e$  are also distinct, and are not contained in  $(\mathfrak{P}/\mathfrak{P}^e)^2$ . But

$$\frac{((x), \mathfrak{P}^2)}{\mathfrak{P}^e} \cdot \left(\frac{\mathfrak{P}}{\mathfrak{P}^e}\right)^{e-2} = \frac{((x), \mathfrak{P}^2)}{\mathfrak{P}^e} \cdot \frac{\mathfrak{P}^{e-2}}{\mathfrak{P}^e} = \frac{(x \mathfrak{P}^{e-2}, \mathfrak{P}^e)}{\mathfrak{P}^e},$$

and

$$\frac{((x), \mathfrak{P}^e)}{\mathfrak{P}^e} \cdot \left(\frac{\mathfrak{P}}{\mathfrak{P}^e}\right)^{e-2} = \frac{(x \mathfrak{P}^{e-2}, \mathfrak{P}^{2e-2}, \mathfrak{P}^e)}{\mathfrak{P}^e} = \frac{(x \mathfrak{P}^{e-2}, \mathfrak{P}^e)}{\mathfrak{P}^e}.$$

So that we have

$$\frac{((x), \mathfrak{P}^2)}{\mathfrak{P}^e} \cdot \left(\frac{\mathfrak{P}}{\mathfrak{P}^e}\right)^{e-2} = \frac{((x), \mathfrak{P}^e)}{\mathfrak{P}^e} \cdot \left(\frac{\mathfrak{P}}{\mathfrak{P}^e}\right)^{e-2},$$

while the first factors of both sides are distinct. This gives an example for the former part of § 5, (i), (ii), if we consider the quotient ring  $\mathfrak{R}/\mathfrak{P}^e$ .

§ 8. Let  $K$  be the quadratic field with discriminant  $d$ ,  $Q$  a rational integer, and  $\mathfrak{R}$  the ring, with *Führer*  $(Q)$ , of  $K$ . Then it is easily shown that the elements of  $\mathfrak{R}$  may be expressed in the form

$$x + y \cdot \frac{Q(d + \sqrt{d})}{2},$$

where  $x, y$  are rational integers.<sup>2</sup> If  $p$  is an odd prime, which divides  $Q$ , the congruence  $b^2 \equiv Q^2 d \pmod{4p}$  is satisfied by  $b \equiv$  either 0 or  $p \pmod{2p}$ . And the ideal

$$\left(p, \frac{-b + Q\sqrt{d}}{2}\right) \quad [b^2 \equiv Q^2 d \pmod{4p}]$$

of  $\mathfrak{R}$  is maximal. Moreover, if this ideal is denoted by  $\mathfrak{P}$ , it can be proved that

<sup>1</sup> For the sake of simplicity, the ideal derived from the principal ideals  $(A), (B), \dots$  will hereafter be denoted by  $(A, B, \dots)$ .

<sup>2</sup> See Weber, *Lehrbuch der Algebra* III, 2nd ed., § 96.

$$\mathfrak{P}^2 = p\mathfrak{P},$$

and also that the principal ideal  $(p)$  is distinct from  $\mathfrak{P}$  and  $\mathfrak{P}^2$ . And, if  $e > 2$ , we have

$$\left(\frac{\mathfrak{P}}{\mathfrak{P}^e}\right)^2 = \frac{\mathfrak{P}^2}{\mathfrak{P}^e} = \frac{p\mathfrak{P}}{\mathfrak{P}^e} = \frac{(p)}{\mathfrak{P}^e} \cdot \frac{(\mathfrak{P})}{\mathfrak{P}^e},$$

while

$$\frac{((p), \mathfrak{P}^2)}{\mathfrak{P}^e} = \frac{(p)}{\mathfrak{P}^e}.$$

If we consider the quotient ring  $\mathfrak{R}/\mathfrak{P}^e$ , this gives an example for the latter part of § 5, (i) (ii).

Examples for case (2) of § 5 can easily be given from the ring  $\mathfrak{R}$ : so that here they are omitted.

November, 1918.

Vol. III, No. 10, Published March 30th, 1919.

Price - - - Yen .45

These Memoirs are on sale at

MARUZEN COMPANY, LTD.

TOKYO, OSAKA, KYOTO, FUKUOKA & SENDAI.

大正八年三月二十七日印刷

大正八年三月三十日發行

編纂兼發行者 京都帝國大學理科大学

印刷者

大久保秀次郎

東京府北豐島郡巢鴨町三丁目十番地

印刷所

株式會社 東京築地活版製造所

東京市京橋區築地二丁目十七番地

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